MOTION IN A STRAIGHT LINE

1. Introduction

1.1 KINEMATICS

The study of the motion of the objects without taking into account the cause of their motion is called kinematics.

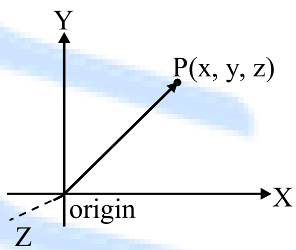
1.2 Frame of reference :

Three mutually perpendicular lines intersecting at a point is called frame of reference. Intersecting point is called the origin and three lines are named as X, Y & Z axes.

1.3 Position vector :

It is a line segment joining the position of in space to the origin of the reference frame directed from origin to particle.

From the figure, Position vector of the points p is $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$



1.4 Rest & Motion :

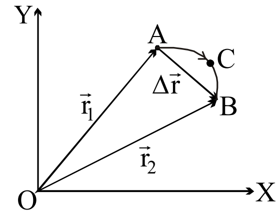
If position vector of a particle in a given reference frame does not change with time then it is said to be at rest with respect to that reference frame, and if its position vector changes with time then it is known as in motion with respect to the given reference frame. The state of rest and motion depends on the frame of reference.

2. Distance & displacement

(i) Distance is the length of the actual path travelled by a particle

(ii) Displacement of a particle is defined as the change in position vector of the particle

(iii) Let us suppose that a particle is moving from the point A to point B through C as shown



(a) If we draw an arrow from the initial position A to the final position B, the vector $\overrightarrow{AB}$ so drawn is called the displacement of the particle in going from A to B.

(b) If $\vec{r}_i = \overrightarrow{OA}$ = position vector of initial position of particle.

$\vec{r}_f$ = position vector of final position = $\overrightarrow{OB}$

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \vec{r}_f - \vec{r}_i$

(c) Distance travelled by the particle length of the curve ACB.

(iv) Distance ≥ |Displacement|

3. Speed

Def : It is the rate of change of distance cover with respect to time covered with the particle and it is a scalar quantity

(i)Instantaneous Speed :

It is the speed of a particle at particular instant. When we say "speed", it usually means instantaneous speed.

Instantaneous speed

$$= \lim_{\Delta t \to 0} \frac{\Delta S}{\Delta t} = \frac{dS}{dt}$$

(ii) Average speed $= \dfrac{\text{Total distance}}{\text{Total time}}$

(iii) Uniform speed :

If during the entire motion magnitude of speed of the body remains same, the body is said to have uniform speed.

(iv) Non-uniform speed :

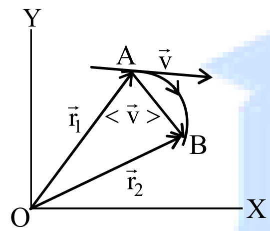
If magnitude of speed changes, the body is said to have non-uniform speed.

4. Velocity

It is defined as rate of change of displacement and it is a vector quantity

(i) Instantaneous Velocity :

It is defined as the velocity at some particular instant. Instantaneous velocity is also called simply velocity.



Instantaneous velocity = $\lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$

(ii) Average Velocity :

Average velocity = $\frac{\text{Total displacement}}{\text{Total time}}$

(iii) Uniform Velocity :

A particle is said to have uniform velocity, if magnitude as well as direction of its velocity remains same and this is possible only when the particles moves in same straight line without reversing its direction.

Special Note :

(a) If a particle moves a distance at speed v_1 and comes back with speed v_2, then

Average speed $v_{av} = \frac{2v_1v_2}{v_1 + v_2}$

& average velocity= 0 [as displacement = 0]

(b) If a particle moves for two equal time-intervals with speed v_1 and v_2 respectively then average speed $v_{av} = \frac{v_1 + v_2}{2}$

(c) Since |displacement| $\leq$ distance, hence |average velocity| $\leq$ average speed i.e. Magnitude of average velocity is always less than or equal to average speed for the same interval of time.

5. Acceleration

It is defined as the rate of change of velocity and it is a vector quantity

(i) Instantaneous acceleration :

It is defined as the acceleration of a body at some particular instant. Instantaneous acceleration

$$= \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

(ii) Average acceleration :

$$\vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$$

* The direction of average acceleration is the direction of the change in velocity vector i.e. $\uparrow\uparrow \Delta \vec{v}$

(iii) Uniform acceleration :

A body is said to have uniform acceleration if magnitude and direction of the acceleration remains constant during particle motion.

Note : If a particle is moving with uniform acceleration, this does not necessarily imply that particle is moving in straight line.

Example : Two dimension projectile motion(parabolic path).

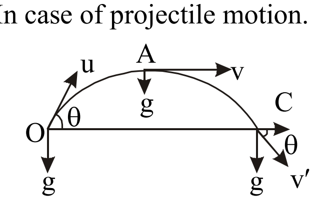
(iv) Non-uniform acceleration :

A body is said to have non-uniform acceleration, if it's magnitude or direction or both, change during motion.

6. Important points regarding acceleration

(i) There is no definite relation between the direction of velocity vector and the direction of acceleration vector i.e. angle between velocity and acceleration may have any value. For Example

(a) In case of projectile motion. (figure)



The angle between acceleration and velocity is $\left(\frac{\pi}{2}+\theta\right)$ at O, $\frac{\pi}{2}$ at A and $\left(\frac{\pi}{2}-\theta\right)$ at C.

(b) For a ball thrown vertically upward, the angle between velocity and acceleration is 180° while for a ball falling downward this angle is 0°, (zero).

(ii) If a body is acted upon by a constant acceleration, its path

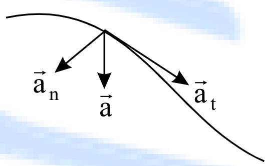
(a) will be a straight line if its initial velocity is along the line of acceleration.

(b) will be a parabola if its initial velocity is making some angle other than zero or 180° with the acceleration.

(iii) If the magnitude of velocity is constant and only its direction changes with time, then acceleration is perpendicular to the velocity vector.

(iv) If an object is moving along a straight line, its acceleration vector is along the line of motion.

(v) In general, the path followed by a particle may be curved. Then net acceleration of the particle has two components.



(a) Tangential acceleration - along the tangent to the path. Tangential acceleration is the rate of change of speed

$$a_t = \frac{d|\vec{v}|}{dt}$$

(b) Normal acceleration - along the normal to the tangent line.

Normal acceleration = v^2/r

where v = speed of the particle ; r = radius of curvature of the path

(vi) For a body moving with uniform acceleration, we have average acceleration = instantaneous acceleration

7. Types of motion

(i) One Dimensional Motion

When a particle moves always in a straight line, its motion is called 1D Motion.

(eg. a ball is thrown vertically upward from the ground.)

(ii) Two dimensional motion

When velocity and acceleration vectors always lie in a single plane (but not collinear) motion is 2D.

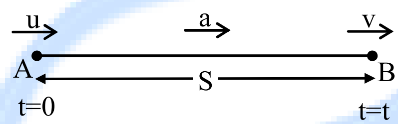
(eg. ball thrown in air at an angle from ground).

(iii) Three dimensional motion

When motion can not be confined in a line or plane it is 3-dimensional motion.

eg. motion of fly in a closed container.

8. Motion in one dimension with constant acceleration



A particle moves from A to B under uniform acceleration with u and v as velocities at A and B respectively. These parameter are related as :

(a) $a = \frac{v-u}{t}$

(b) $v = u + at$

(c) average velocity

$$v_{avg} = \frac{u+v}{2}$$

(d) $S = v_{avg}\,t$ or $S = ut + \frac{1}{2}at^2$

(e) $v^2 = u^2 + 2as$

(f) Displacement of particle in n^{th} second of its motion is

$$S_n = u + \frac{1}{2}a(2n-1)$$

9. Motion under gravity (one dimension)

(i) The most important example of motion along a straight line with constant acceleration is motion under gravity. In case of motion under gravity unless stated it is taken for granted that.

(a) The acceleration is constant, i.e. $|\vec{a}| = |\vec{g}|$ = 9.8 m/s^2 and directed vertically downwards.

(b) The motion is in vacuum i.e. viscous force or thrust of the medium has no effect on the motion.

(ii) Ball is dropped from height

If a small ball is dropped from a tower of height H then

(A) time taken to reach the ground is

$$t = \sqrt{\frac{2H}{g}}$$

$$(S = ut + \frac{1}{2}at^2,\ -H = 0 - \frac{1}{2}gt^2,\ t = \sqrt{\frac{2H}{g}})$$

(B) Speed of ball when it reach the ground

$$v = \sqrt{2gH}$$

$$(v^2 = u^2 + 2as,\ v^2 = 0 + 2gH)$$

(C) Important point :

(a) If the body is dropped from a height H, as in time t, it has fallen a distance h from its initial position, the height of the body from the ground will be h' = H – h, with $h = 1/2\ gt^2$.

(b) As $h = (1/2)\ gt^2$ i.e. $h \propto t^2$, distance fallen in time t, 2t, 3t etc. will be in the ratio of $1^2 : 2^2 : 3^2$: -------i.e. square of integers.

(c) The distance fallen in n^{th} sec., $h_n - h_{n-1} = (1/2)\ g(n)^2 - (1/2)\ g(n-1)^2 = (1/2)\ g(2n-1)$

So distance fallen in Ist, IInd, IIIrd sec. will be in the ratio 1 : 3 : 5 i.e. odd integers only

(iii) Body is projected vertically up :

A. If a particle is projected up with velocity u, then

(a) maximum height reached by the particle

$$H = \frac{u^2}{2g}$$

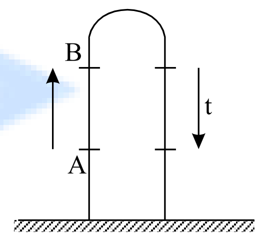
(b) Time taken to reach the maximum height

$$= t = \frac{u}{g} = \sqrt{\frac{2H}{g}}$$

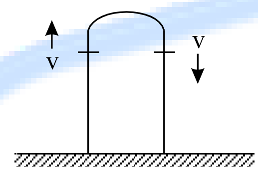
(c) Time taken to come back at the point of projection (Time of flight) $T = \frac{2u}{g}$

B. Important points

(a) A ball thrown vertically up takes the same time to go up and come down and it is true for any part of its motion



(b) A particle has the same speed at a point on the path while going vertically up and down.



(c) If a particle is dropped from a height H above the ground, then

(i) velocity of the particle when it reaches the ground $v = \sqrt{2gH}$

(ii) time taken to reach the ground

$$t = \sqrt{\frac{2H}{g}}$$

(d) Whenever a ball is dropped, its initial velocity is equal to the velocity of the body where from it is being dropped. Just after dropping, acceleration of the ball will be equal to free fall acceleration i.e. gravitational acceleration g.

(e) If we consider constant retarding force due to air resistance, then the ball takes less time to reach the highest position and larger time to reach the ground as compared to that in the absence of air resistance.

10. One dimensional motion with variable acceleration

(i) Acceleration may vary with time or position or velocity. Hence variable acceleration can be divided into three parts.

Variable Acceleration
- Time dependent acceleration
- Position dependent acceleration
- Velocity dependent acceleration

(ii) If a particle is moving along x-axis, then Instantaneous velocity

$$\frac{dx}{dt} = v \qquad ...(i)$$

Instantaneous acceleration

$$\frac{dv}{dt} = a \qquad ...(ii)$$

Note : While using equation (i) and (ii), we should put known values with proper sign and unknown values should not be touched.

(iii) (A) To solve problems involving time dependent acceleration :

Step(1) Let acceleration a = f(t), where f is a function of time t

$$\text{write } a = \frac{dv}{dt} = f(t) \qquad ...(1)$$

Step(2) Integrate the above equation to get v as a function of time t

$$dv = f(t)dt$$

$$v = \int dv = \int f(t)dt + A \qquad ...(2)$$

Where A is the integration constant whose value can be found from the initial condition

Step(3) Write $v = \frac{dx}{dt}$ in equation (2) and integrate it to get position coordinate x.

(B) To solve problem in involving position dependent acceleration :

Step(1) Let acceleration a = g(x), where g is a function of position x.

$$\text{Write} \quad a = v\frac{dv}{dx} = g(x) \quad ...(1)$$

$$\therefore \quad a = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx}$$

Step(2) Integrate the above equation (1) to get v as a function of position x.

$$vdv = g(x)\,dx$$

$$\int vdv = \frac{v^2}{2} = g(x)\,dx + B \qquad ...(2)$$

Where value of B can be determined from the initial condition

Step(3) Write $v = \frac{dx}{dt}$ in equation (2) and integrate it to get position co-ordinate x.

(C) To solve problems involving velocity dependent acceleration :

Step(1) Let acceleration a = f(v), where f is a function of velocity v

$$\text{Write } a = \frac{dv}{dt} = f(v)$$

Step(2) Integrating

$$\int \frac{dv}{f(v)} = dt = t + C$$

Step(3) Write $v = \frac{dx}{dt}$ in the obtained equation and integrate it to get position co-ordinate x.

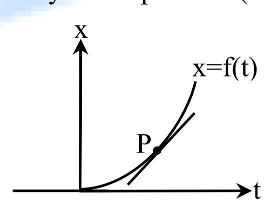
11. Graph (one dimensional motion)

From various graphs we can get the information about the following quantities.

Displacement, Distance, Instantaenous velocity, average velocity, instantaneous acceleration, average acceleration, nature of the motion.

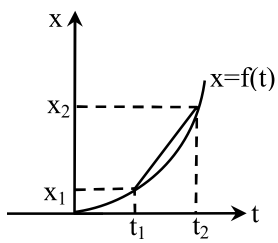
(i) Position time graph :

(a) Slope of the tangent at any point on the graph represents the instantaneous velocity of the particle. (v = dx/dt)



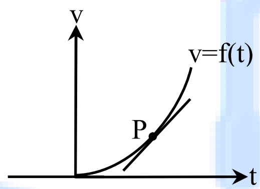
(b) Slope of the chord joining any two points on the graph represents the average velocity, between those two points(time interval).

$$v_{avg} = \frac{x_2 - x_1}{t_2 - t_1}$$



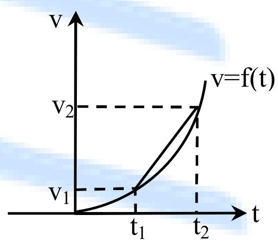
(ii) Velocity time graph :

(a) Slope (a = dv/dt) of the tangent at any point on the graph represents the instantaneous acceleration of the particle.



(b) Slope of the chord joining any two points on the graph represents the average acceleration between those two points(time interval).

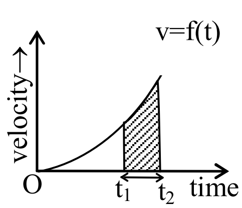
$$a_{avg} = \frac{v_2 - v_1}{t_2 - t_1}$$



(c) Area bounded by the graph and the time axis between a given time interval represents the displacement of the particle in that time interval.

$$\text{As, } v = \frac{ds}{dt} \Rightarrow ds = v dt$$

$$\therefore s = \int_{t_1}^{t_2} v\,dt$$



Note : If graph cuts the time axis then area lies on the positive side is positive and the area lies on the negative side is negative.

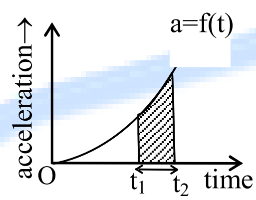
(d) Sum of the magnitudes of the areas represents the distance moved.

(iii) Acceleration time graph :

(a) Area bounded by the graph and the time axis between a given time interval represents change in velocity of the particle in that time interval.

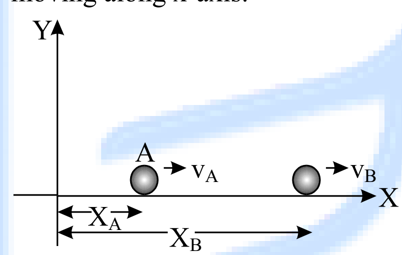
$$\text{As, } a = \frac{dv}{dt} \Rightarrow dv = a\,dt$$

$$\therefore v_2 - v_1 = \int_{t_1}^{t_2} v\,dt$$



12. Relative motion in one dimension

(i) Let us suppose two particles A and B are moving along x-axis.



Let x_A = displacement of A w.r.t. the fixed origin O

x_B = displacement of B w.r.t. the fixed origin O

v_A = velocity of A w.r.t. the fixed origin O

v_B = velocity fo B w.r.t. the fixed origin O

a_A = acceleration of A w.r.t. the fixed origin O

a_B = acceleration of B w.r.t. the fixed origin O

Then

(a) The relative displacement of B w.r.t. A is defined as $x_{BA} = x_B - x_A$ ...(1)

(b) The relative velocity of B w.r.t. A is defined as $v_{BA} = v_B - v_A$ ...(2)

(c) The relative acceleration of B w.r.t. A is defined as $a_{BA} = a_B - a_A$ ...(3)

(ii) In case of relative motion, the fundamental equations of kinematics in one dimension are modified as

$v_{BA} = u_{BA} + a_{BA}t$ ....(4 - a)

$x_{BA} = u_{BA}t + \frac{1}{2}a_{BA}t^2$ ....(4 - b)

$v_{BA}^2 = u_{BA}^2 + 2\,a_{BA}\,x_{BA}$ ....(4 - c)

Note : While using all of the above equations [equations (1) to (4)], we must put the known values with proper sign and unknown value must not be touched. Unknown physical quantity will be obtained with proper sign.

(iii) Important Results :

(a) If two bodies are moving along the same line in same direction with velocities of magnitudes V_A and V_B relative to earth, the velocity of B relative to A will be given by $V_{BA} = V_B - V_A$. If it is positive then the direction of V_{BA} is that of B and if it is negative then the direction of V_{BA} is opposite to that of B.

(b) However, if the bodies are moving towards or away from each other, as direction of V_A and V_B are opposite, velocity of B relative to A will have magnitude $V_{BA} = V_B - (-V_A) = V_B + V_A$ and directed towards A or away from A respectively.

(c) In dealing the motion of two bodies relative to each other $\vec{v}_{rel}$ is the difference of velocities of two bodies, if they are moving in same direction and is the sum of two velocities if they are moving in opposite direction.

(d) A boy running on a rail road car :

$\vec{v}_{rel}$ = velocity of boy relative to car

$\vec{v}_c$ = velocity of car relative to ground

$\vec{v}_b$ = velocity of boy relative to ground

then $\vec{v}_b = \vec{v}_{rel} + \vec{v}_c$

Case (I) $\vec{v}_{rel} \uparrow\uparrow \vec{v}_c$, $v_b = v_{rel} + v_c$

Case(II) $\vec{v}_{rel} \uparrow\downarrow \vec{v}_c$, $v_b = v_{rel} - v_c$

SOLVED EXAMPLES

Ex. 1 The displacement of a particle undergoing rectilinear motion along the x-axis is given by $x = (2t^3 - 21t^2 + 60t + 6)$ m. Find the acceleration of the particle when its velocity is zero.

Sol. $x = 2t^3 - 21t^2 + 60t + 6$

velocity $\frac{dx}{dt} = 6t^2 - 42t + 60$

acceleration $\frac{d^2x}{dt^2} = 12t - 42$

when velocity, $v = \frac{dx}{dt} = 0$, then

$0 = 6t^2 - 42t + 60 \quad \Rightarrow \quad t = 5 \text{ or } 2$

Now acceleration $a = \frac{d^2x}{dt^2} = 12t - 42$

$\therefore$ acceleration $= (12 \times 5) - 42 = 18 \text{ m/s}^2$

or acceleration $= (12 \times 2) - 42 = -18 \text{ m/s}^2$

Ex. 2 A truck starts from rest with an acceleration of 1.5 metre/sec^2 while a car 150 metre behind starts from rest with an acceleration of 2 metre/sec^2. How long will it take before both the truck and car to be side by side, and how much distance is travelled by each ?

(A) 2.45 sec (B) 5 sec

(C) 24.5 sec (D) 5.3 sec

Sol. Let x be the distance travelled by the truck when both truck and car are side by side. The distance travelled by the car will be (x + 150) as the car is 150 metre behind the truck. Applying the formula

$s = ut + (1/2)\, a t^2$, we have

$x = 1/2 \times (1.5)\, t^2$ ....(1)

and $(x + 150) = (1/2) \times (2)\, t^2$ ....(2)

Here t is the common time.

From eqs. (1) and (2) we have

$$= \frac{x + 150}{x} = \frac{2}{1.5}$$

Solving we get x = 450 metre (truck) and x + 150 = 600 metre (car).

Substituting the value of x in eq. (1),

we get $450 = 1/2\, (1.5)\, t^2$

$\therefore \quad t = \sqrt{\frac{450 \times 2}{1.5}} = \sqrt{600} = 24.5$ sec.

Hence correct answer is (C)

Ex. 3 A point moving with constant acceleration from A to B in the straight line AB has velocities u and v at A and B respectively. Find its velocity at C, the mid-point of AB. Also show that if the time from A to C is twice that from C to B, then v = 7u.

Sol. Let the particle move with a constant acceleration a. At point A its velocity is u while at point B its velocity is v. Let the distance between A and B be s, then -

$v^2 = u^2 + 2as$ ...(1)

If v_1 be the velocity of the point at C, then

$$v_1^2 = u^2 + 2a\,(s/2) \qquad ...(2)$$

($\therefore$ distance AC = s/2)

From equations (1) and (2), we have

$$v^2 = u^2 + 2(v_1^2 - u^2)$$

or $v^2 = 2v_1^2 - u^2$

$\therefore$ $2v_1^2 = v^2 + u^2$

or $$v_1 = \sqrt{\left(\frac{v^2 + u^2}{2}\right)} \qquad ...(3)$$

Let t be the time taken from C to B. As time taken between A and C is twice than that of C to B hence time taken between A and C is 2t. Thus total time between A and B is 3t. Using the formula $s = \left(\frac{u+v}{2}\right)$ t, we have

$$s = \left(\frac{u+v}{2}\right) . 3t \qquad ...(4)$$

and $$\frac{s}{2} = \left(\frac{v_1 + v}{2}\right) . t \qquad ...(5)$$

From these equations, we get

$$\frac{3}{2}\,(u + v)\,t = (v_1 + v)\,t$$

or $3u + v = 2v_1 \qquad ...(6)$

Substituting the value of v_1 in equation (6) from equation (3), we get

$$3u + v = 2\sqrt{\left(\frac{v^2 + u^2}{2}\right)}$$

Squaring and solving we get

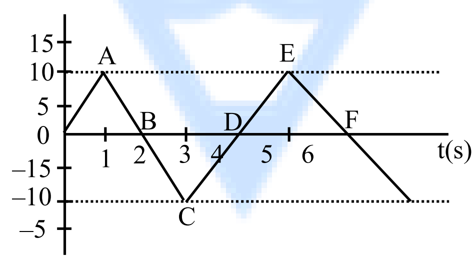
$$9u^2 + v^2 + 6uv = 2u^2 + 2v^2$$

or $v^2 - 6uv - 7u^2 = 0$

$(v + u)(v - 7u) = 0$ but $u + v \neq 0$

$(v - 7u) = 0$ or $v = 7u$.

Ex.4 From the velocity - time graph of a particle given in figure, describe the motion of the particle qualitatively in the interval 0 to 4s. Find (a) the distance travelled during first two seconds, (b) during the time 2s to 4s, (c) during the time 0 to 4s, (d) displacement during 0 to 4s, (e) acceleration at t = 1/2 s and (f) acceleration at t = 2 s.



Sol. At t = 0, the particle is at rest, say at the origin. After that the velocity is positive, so that the particle moves in the positive x direction. Its speed increases till 1 second when it starts decreasing. the particle continues to move further in positive x direction. At t = 2s, its velocity is reduced to zero, it has moved through a maximum positive x distance. Then it changes its direction, velocity being negative, but increasing in magnitude. At t = 3s velocity is maximum in the negative x direction and then the magnitude starts decreasing. It comes to rest at t = 4 s.

(a) Distance during 0 to 2 s

$= \text{Area of OAB} = \frac{1}{2} \times 2\text{s} \times 10 \text{ m/s} = 10 \text{ m}$

(b) Distance during 2 to 4s = Area of BCD = 10 m. The particle has moved in negative × direction during this period.

(c) The distance travelled during 0 to 4s
= 10 m + 10m = 20m.

(d) displacement during 0 to 4s
= 10 m + (–10 m) = 0.

(e) at t = 1/2s acceleration = slope of line OA
$= 10 \text{ m/s}^2$.

(f) at t = 2 s acceleration= slope of line ABC
$= -10 \text{ m/s}^2$.

Ex. 5 A particle beginning from rest, travels a distance S with uniform acceleration and immediately after travels a distance of 3S with uniform speed followed by a distance 5S with uniform deceleration, and comes to rest.

Find the ratio of average speed to the maximum speed of the particle.

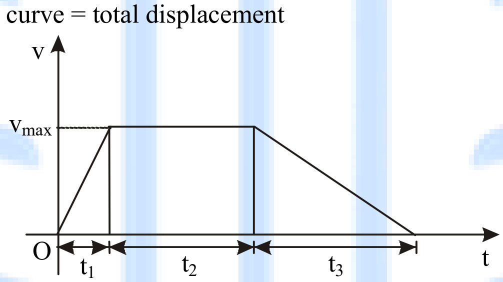
Sol. Let the maximum speed = v_{max}.

total time of acceleration = t_1

total time of uniform velocity = t_2

total time of deceleration = t_3

∴ Area under the v/t curve = total displacement



$\therefore \quad S = \frac{v_{max} t_1}{2}$

$3S = v_{max}\, t_2$

$5S = \frac{v_{max} t_3}{2}$

$\therefore\ v_{max}(t_1 + t_2 + t_3) = 2S + 3S + 10S = 15\ S$

Now average speed $\bar{v} = \frac{S + 3S + 5S}{t_1 + t_2 + t_3}$

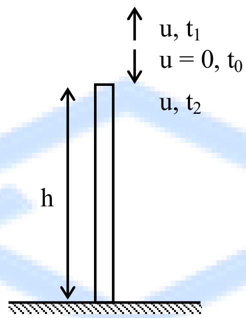
Hence required ratio

$= \frac{\bar{v}}{v_{max}} = \frac{S + 3S + 5S}{v_{max}(t_1 + t_2 + t_3)}$

$= \frac{9S}{15S} = \frac{3}{5}$

Ex.6 When a particle is projected upward with speed u from the top of a tower, it reaches the ground in time t_1. When it is projected downward with the same speed, it reaches the ground in time t_2. How long does it take to reach the ground if it is just dropped.

Sol. Let the height of the tower be h metre if we take the upward direction as (+)ve direction, then



from $S = ut + \frac{1}{2}at^2$

$$-h = ut_1 - \frac{1}{2}gt_1^2 \qquad ....(1)$$

$$-h = -ut_2 - \frac{1}{2}gt_2^2 \qquad ....(2)$$

$$-h = 0 - \frac{1}{2}gt_0^2 \qquad ....(3)$$

where t_0 is the required time.

Multiplying equation (1) by t_2 and equation (2) by t_1 and then adding, we get

$$-h(t_2 + t_1) = -\ gt_1\, t_2\, (t_1 + t_2)$$

$$\Rightarrow\ h = \frac{1}{2}gt_1\, t_2 \qquad ....(4)$$

from equation (3) and equation (4), we get

$$t_0^2 = t_1\, t_2 \Rightarrow t_0 = \sqrt{t_1 t_2}$$

Ex. 7 From the foot of a tower 90 m high a stone is thrown up so as to just reach the top of the tower. Two second later another stone is dropped from the top of the tower. When and where two stones meet.

Sol. Let the two stones meet t seconds after the projection of the first particle. The sum of the distance moved by the particles is 90 meters i.e., $h_1 + h_2 = 90$ ....(1)

Let u be the velocity of projection of the first particle. As it reaches only up to the top of the tower, its velocity becomes zero, so

$v^2 = u^2 - 2\,g\,h$ or $0 = u^2 - 2\,g.\,90$

or $u^2 = 180\,g$, $u = \sqrt{180 \times 9.8} = 42$ m/sec.

Now $h_1 = 42\,t - (1/2)\,.\,9.8 \times t^2$ and

$h_2 = (1/2)\,9.8\,(t - 2)^2$ ....(2)

Substituting these values in eq. (1), we get

$42\,t - 4.9\,t^2 + 4.9\,(t - 2)^2 = 90$

or $42\,t - 19.6\,t + 19.6 = 90$

$\Rightarrow\ t = \frac{70.4}{22.4} = \frac{22}{7} = 3\frac{1}{7}$ sec,

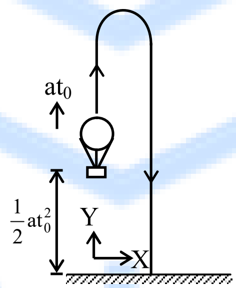
$\therefore h_2 = \frac{9.8}{2} \times \left(\frac{22}{7} - 2\right)^2$

$= 4.9 \times \frac{64}{49} = 6.4$ metre,

$h_1 = 90 - 6.4 = 83.6$ metre

Ex.8 A balloon starts rising upward with constant acceleration a and after t_0 second a packet is dropped from it which reaches the ground after t second. Determine the value of t.

Sol. Assuming origin at the ground we have



$y_0 = +\frac{1}{2}at_0^2;$

$v_0 = +at_0;$

$a = -g;\ y = 0$

Substituting the above values in the equation

$y = y_0 + v_0t + \frac{1}{2}at^2$

we get $0 = \frac{1}{2}at_0^2 + at_0t - \frac{1}{2}gt^2$

or $t^2 - \frac{2at_0t}{g} - \left(\frac{a}{g}\right)t_0^2 = 0$

Solving the quadratic equation, we get

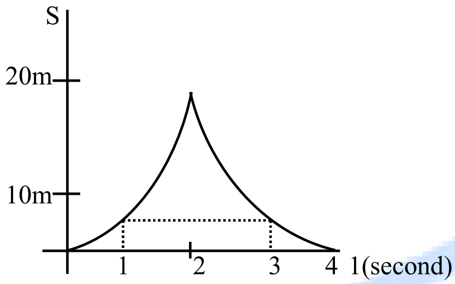
$$t = \frac{at_0}{g}\left[1 + \sqrt{1 + \frac{g}{a}}\right]$$

Ex.9 A ball is dropped from a height of 19.6 m above the ground. It rebounds from the ground and raises itself up to the same height. Take the starting point as the origin and vertically downward as the positive x-axis. Draw approximate plots of x versus t, v versus t and a versus t. Neglect the small interval during which the ball was in contact with the ground.

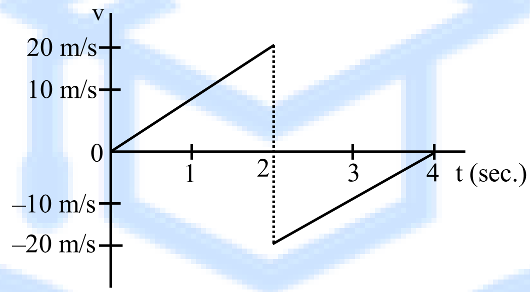
Sol. Since the acceleration of the ball during the contact is different from 'g', we have to treat the downward motion and the upward motion separately. For the downward motion

$a = g = 9.8\ m/s^2$,

x = ut + at^2 = (4.9 m/s^2)t^2.



The ball reaches the ground when x = 19.6 m. This gives t = 2s. After that it moves up, x decreases and at t = 4s, x becomes zero, the ball reaching the initial point.



We have at	t = 0,	x = 0
	t = 1s,	x = 4.9 m
	t = 2s	x = 19.6 m
	t = 3s,	x = 4.9 m
	t = 4s,	x = 0

Velocity : During the first two seconds,

v = u + at = (9.8 m/s^2)t

at t = 0 v = 0

at t = 1s, v = 9.8 m/s

at t = 2s, v = 19.6 m/s

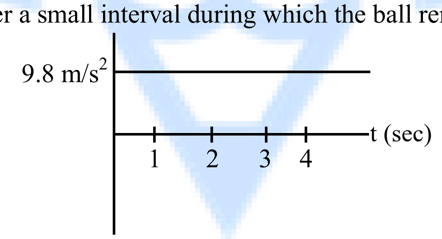
During the next two seconds the ball goes upward, velocity is negative, magnitude decreasing and at t = 4s, v = 0. Thus,

at t = 2s, v = – 19.6 m/s

at t = 3s, v = – 9.8 m/s

at t = 4s, v = 0

At t = 2s there is an abrupt change in velocity from 19.6 m/s to – 19.6 m/s. In fact this change in velocity takes place over a small interval during which the ball remains in contact with the ground.



Acceleration : The acceleration is constant 9.8 m/s^2 throughout the motion (except at t = 2s)

Ex. 10 A rocket is fired vertically up from the ground with a resultant vertical acceleration of 10m/s^2. The fuel is finished in 1 minute and it continues to move up.

(a) the maximum height reached.

(b) After how much time from then will the maximum height be reached

(Take $g = 10\ m/s^2$)

(A) 36km, 1 min (B) 6km, 1 min

(C) 36km, 1 sec (D) 36 km, 1 sec

Sol. (a) The distance travelled by the rocket during burning interval (1 minute = 60 s) in which resultant acceleration is vertically upwards and is $10\ m/s^2$ will be

$h_1 = 0 \times 60 + (1/2) \times 10 \times 60^2 = 18000$ m ...(1)

And velocity acquired by it will be

$v = 0 + 10 \times 60 = 600$ m/s ...(2)

Now after 1 minute the rocket moves vertically up with initial velocity of 600 m/s and acceleration due to gravity oppose its motion.

So, it will go to a height h_2 till its velocity becomes zero such that

$$0 = (600)^2 - 2gh_2$$

$\Rightarrow h_2 = 18000$ m [as $g = 10 m/s^2$] ..(3)

So from eq. (1) and (3) the maximum height reached by the rocket from the ground.

$H = h_1 + h_2 = 18 + 18 = 36$ km (b) As after burning of fuel the initial velocity from Eq. (2) is 600 m/s and gravity opposes the motion of rocket, so from 1st equation of motion time taken by it to reach the maximum height (for which v = 0)

$0 = 600 - gt$, i.e.t = 60 s

after finishing of fuel, the rocket goes up for 60 sec i.e., 1 minute more.

Hence correct answer is (A)

Ex.11 A car, starting from rest, starts moving with an acceleration $a = \sqrt{t}$. At the same instant a truck passes that point with a velocity 4 m/s in the same direction. After how much distance, the car overtakes the truck ?

Sol. Let the car overtakes the truck in n seconds then distance d travelled by the vehicles in n seconds must be the same. That is

for truck : d = 4n

for car : $d = \int_0^n v\, dt$

Now velocity of the car as a function of time is

$$v = \int_0^t a\ dt = \int_0^t \sqrt{t}\ dt = \frac{t^{3/2}}{3/2} \quad \text{(given } u = 0\text{)}$$

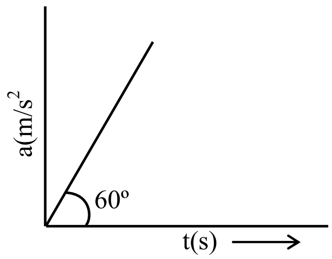
$$\therefore \text{ for car } d = \int_0^n t^{3/2}\, dt$$

$$= \frac{2}{3} \times \frac{2}{5} \times \left[t^{5/2}\right]_0^n = \frac{4}{15}\ n^{5/2}$$

Equating d (truck) = d (car), we get

$4n = \frac{4}{15}\ n^{5/2}$ or $n = (15)^{2/3}$ and distance travelled is $d = 4n = 4(15)^{2/3}$

Ex.12 The acceleration of an object as a function of time is shown is figure. If at t = 0, the velocity of the particle is u = 0, then



(i) What is the object's velocity at any latter time ?

(ii) What is the distance travelled by the particle in time t ?

Sol. In this problem the acceleration is not constant, therefore, v = u + at etc. formulas can not be applied. From figure it is given that

slope = $\frac{a}{t}$ = tan 60° = $\sqrt{3}$

$\therefore$ a = $\sqrt{3}$ t

(i) from a = $\frac{dv}{dt}$ we write dv = adt = $\sqrt{3}$ t dt

or $\int dv = \int \sqrt{3}\, t\, dt$

$\therefore$ v – u = $\sqrt{3}\,\frac{t^2}{2}$ but u = 0 given, thus

v = $\frac{\sqrt{3}}{2}t^2$

(ii) from v = $\frac{dx}{dt}$, we write

dx = v dt = $\frac{\sqrt{3}}{2}\int t^2\, dt$

Integrating $\int dx = \frac{\sqrt{3}}{2}\int t^2 dt$

or x – x_0 = $\frac{\sqrt{3}}{2}\left(\frac{t^3}{3}\right) = \frac{\sqrt{3}t^3}{6}$

Thus distance travelled in time t is

s = $\sqrt{3}\, t^3/6$

Ex.13 A car is moving with a velocity v_0. The engine of the car suddenly stops and as a result a deceleration dv/dt = – kv^3 acts on the car where k is a constant and v is instantaneous velocity. What will be car's velocity after t seconds (take t = 0, just when engine stops).

Sol. Given $\frac{dv}{dt} = -kv^3$

$\therefore \frac{dv}{v^3} = -k\, dt$ on integrating

$\int_{v_0}^{v}\frac{dv}{v^3} = -k\int_0^t dt$

$$\Rightarrow \left[-\frac{1}{2v^2}\right]_{v_0}^{v} = -kt$$

or $\dfrac{1}{2v_0^2} - \dfrac{1}{2v^2} = -kt$

or $\dfrac{1}{v^2} = \dfrac{1+2kt\,v_0^2}{v_0^2}$

Therefore $v = \dfrac{v_0}{\sqrt{1+2kt\,v_0^2}}$

Ex.14 A particle starts from the origin with an initial velocity of 2m/s along the negative x-axis and with a constant acceleration of 10 m/s^2 along the positive x-axis. Find the instantaneous velocity and displacement of the particle as a function of time -

Sol. Given that $a = \dfrac{dv}{dt} = +10$

$\Rightarrow \int dv = \int 10\,dt$

$\Rightarrow v = 10t + C_1$

At $t = 0$, $v = -2m/s \Rightarrow C_1 = -2$

$\therefore v = 10t - 2$

$\dfrac{dx}{dt} = 10t - 2$

$\int dx = \int (10t-2)dt$

$x = 5t^2 - 2t + C_2$

At $t = 0$, $x = 0 \Rightarrow C_2 = 0$

$\therefore x = 5t^2 - 2t$